POSITIVE INCREMENT

Definition 1. *The **Euler Totient Function** is defined for $n \geq 1$ as the number of positive integers not exceeding n which are relatively prime to n.*

That is to say:

$$\varphi(n) := \sum_{\substack{k=1 \\ (k,n)=1}}^{n} 1. \tag{1}$$

Theorem 1.

$$\limsup_{n\to\infty} \frac{\varphi(n)}{n} = 1. \tag{2}$$

Proof. First notice that by definition $\frac{\varphi(n)}{n} \leq 1$. Also, if we consider the sequence p_k of the prime numbers in growing order, we have:

$$\frac{\varphi(n)}{n} = \frac{\varphi(p_k)}{p_k} = \frac{p_k - 1}{p_k} = 1 - \frac{1}{p_k}$$

therefore:

$$\lim_{k\to\infty} \frac{\varphi(p_k)}{p_k} = \lim_{k\to\infty} 1 - \frac{1}{p_k} = 1$$

and the proof is complete. □