Definition 1. *The Riemann Zeta function is defined as*

$$\zeta(s) := \sum_{n=1}^{\infty} \frac{1}{n^s} \tag{1}$$

for $Re(s) > 1$.

Theorem 1. *The Riemann zeta function satisfies the equation*

$$\zeta(1-s) = 2(2\pi)^{-s} \cos\left(\frac{\pi}{2}s\right)\Gamma(s)\zeta(s) \tag{2}$$

for $s \neq 0, 1$.

Proof. This result can be obtained easily starting from the first functional equation:

$$\zeta(s) = 2(2\pi)^{s-1} \sin\left(\frac{\pi}{2}s\right)\Gamma(1-s)\zeta(1-s) \tag{3}$$

this one requires instead some work, a complete proof can be found on our site.

Changing s into $1-s$ in 3 we have:

$$\begin{aligned}
\zeta(1-s) &= 2(2\pi)^{1-s-1} \sin\left(\frac{\pi}{2}(1-s)\right)\Gamma(1-(1-s))\zeta(1-(1-s)) \\
&\Downarrow \\
\zeta(1-s) &= 2(2\pi)^{-s} \sin\left(\frac{\pi}{2} - \frac{\pi}{2}s\right)\Gamma(s)\zeta(s) \qquad (4) \\
&\Downarrow \\
\zeta(1-s) &= 2(2\pi)^{-s} \cos\left(\frac{\pi}{2}s\right)\Gamma(s)\zeta(s)
\end{aligned}$$

which is exactly 2. □